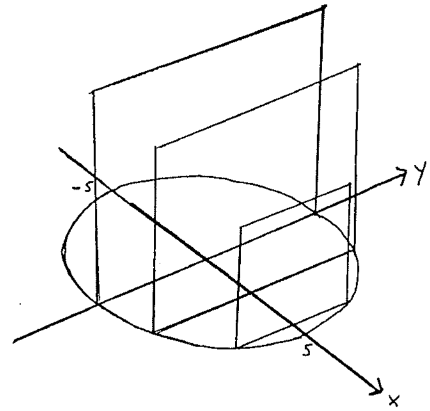
AP Calculus Class Examples Day 1 --Volumes of Solids of Known Cross Sections

1. The base of a solid is the region enclosed by a circle centered at the origin with a radius of 5 inches. Find the volume of the solid if all cross sections perpendicular to the x-axis are squares.



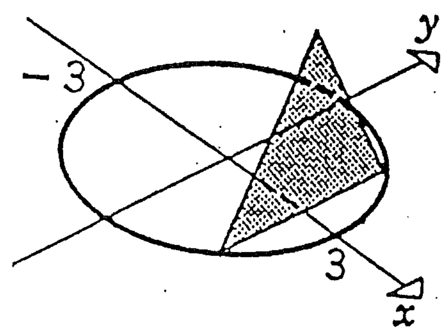
$x^2+y^2=25$

$y^2=25-x^2$

$y=\pm\sqrt{25-x^2}$ $\to$ $f(x)=\sqrt{25-x^2}$, $g(x)=-\sqrt{25-x^2}$

$$V=\int_{-5}^{5}(f(x)-g(x))^2\,dx \approx 666.667$$

2. The base of a solid is the circle centered at the origin with a radius of 3 inches. Find the volume of the solid if all cross sections perpendicular to the x-axis are equilateral triangles.

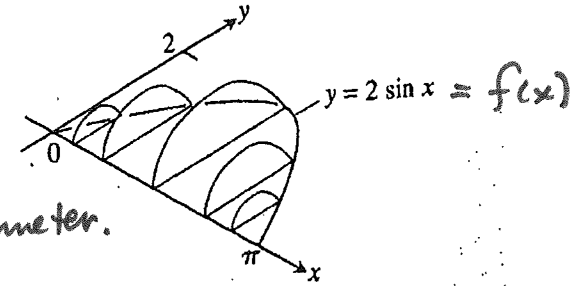


$x^2+y^2=9$

$y=\pm\sqrt{9-x^2}$ $\to$ $f(x)=\sqrt{9-x^2}$, $g(x)=-\sqrt{9-x^2}$

$$V=\int_{-3}^{3}(f(x)-g(x))^2\,dx = 144$$

3. A mathematician has a paperweight made so that its base is the shape of the region between the x-axis and one arch of the curve y = 2 sin x. Each cross-section perpendicular to the x-axis is a semicircle whose diameter runs from the x-axis to the curve. Find the volume of the paperweight.

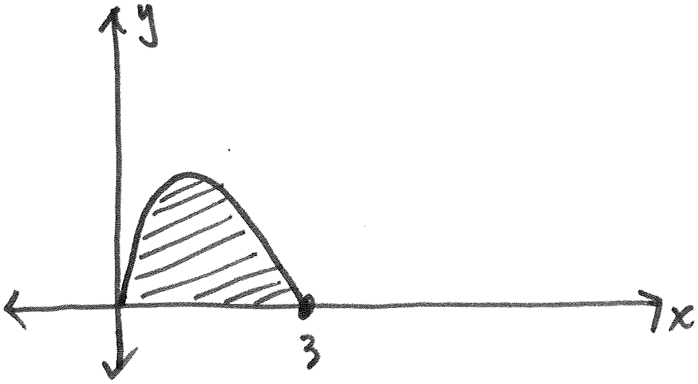


$A(\text{semi-circle}) = \frac{1}{2} \cdot \pi r^2, \quad r = \frac{1}{2}$ the diameter.

$$V = \frac{1}{2}\pi \int_0^{\pi} \left(\frac{f(x)}{2}\right)^2 dx \approx 2.467$$

4. The base of a solid is the region in the first quadrant bounded by the graph of $y = 3x^{\frac{1}{2}} - x^{\frac{3}{2}} = f(x)$ and the x-axis. Cross sections perpendicular to the x-axis are isosceles right triangles, with one leg in the xy-plane. What is the volume of the solid?

$\hookrightarrow A = \frac{1}{2}s^2 = \frac{1}{2}(f(x))^2$



$$V = \frac{1}{2}\int_0^3 (f(x))^2\, dx$$

$$\approx 3.375$$

AP Calculus Classwork – Volumes of Solids of Known Cross Sections $\rightarrow \frac{x^2}{4} + \frac{y^2}{9} = 1$

1. The base of a solid is the elliptical region with boundary curve $9x^2 + 4y^2 = 36$. Cross sections perpendicular to the x-axis are isosceles right triangles with hypotenuse in the base. Find the volume of the solid.

$\hookrightarrow A = \frac{1}{2}s^2 = \frac{1}{2}(f(x) - g(x))^2$

$9x^2 + 4y^2 = 36$

$\frac{4y^2}{4} = \frac{36 - 9x^2}{4}$

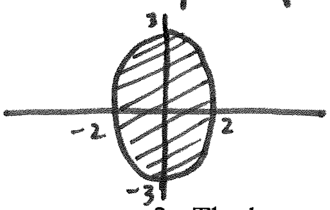
$y = \pm\sqrt{\frac{36 - 9x^2}{4}}$

$f(x) = \sqrt{\frac{36 - 9x^2}{4}}$

$g(x) = -\sqrt{\frac{36 - 9x^2}{4}}$

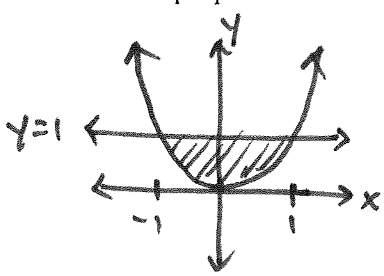
$V = \frac{1}{2}\int_{-2}^{2}(f(x) - g(x))^2\,dx$

$= 48$



2. The base of a solid is a region bounded by the curves $\overset{f(x)}{y = x^2}$ and $\overset{g(x)}{y = 1}$. Cross sections perpendicular to the x-axis are semicircles. Find the volume of the solid.

$A(\text{semi}) = \frac{1}{2}\pi r^2$, $r = \frac{1}{2}$ diameter



$V = \frac{1}{2}\pi\int_{-1}^{1}(g(x) - f(x))^2\,dx \approx 1.675$ or 1.676

3. The base of a solid is the region bounded by the graphs of $x^2 = 16y$ and $y = 2$. Cross sections perpendicular to the x-axis are rectangles whose height is twice that of the side in the xy-plane. Find the volume of the solid.

$f(x) = y = \frac{x^2}{16}$

$g(x) = 2$

$A(\text{rect}) = b \cdot 2b = 2b^2$, $b = g(x) - f(x)$

$V = 2\int_{-4}^{4}(g(x) - f(x))^2 \approx 8.533$